Statistical Mechanics: Problems 11.1

1. Consider N independent, identical harmonic oscillators of frequency ω. Using classical statistical mechanics, calculate
   (a) the average energy
   (b) the specific heat
2. Particles of an ideal gas, enclosed in a volume V, have velocities close to the speed of light c. Their energy has to be calculated relativistically. In the limit of massless particles (e.g., photons) which travel at the speed of light, the energy becomes

$$E = \sum_{i=1}^{N} c|\vec{p}_i|$$

   Evaluate the partition function and the free energy in the canonical ensemble.
   Hint: Use spherical polar coordinates to integrate in momentum space.
3. Consider an electron inside a box of dimensions $L_x \times L_y \times L_z$. There is a uniform electric field $\mathcal{E}$ along the x-axis, and earth's gravitational field along z-axis. Treat the electron as a classical particle within canonical ensemble. Calculate the average energy of the electron at a temperature T. What is the probability of finding the electron inside a cubical volume $\Delta \times \Delta \times \Delta$ located at $(\frac{L_x}{2}, \frac{L_y}{2}, \frac{L_z}{2})$ where Δ is an infinetisimally small length?
4. A classical rigid rotor is free to rotate in any direction, and its only energy is rotational kinetic energy, given by

$$E = \frac{p_\theta^2}{2I} + \frac{p_\phi^2}{2I\sin^2\theta},$$

   where I is the moment of inertia and p_θ, p_ϕ are angular momenta corresponding to two kinds of rotations. Evaluate the partition function and average energy in the canonical ensemble.
5. Consider a gas of atoms of mass m each, and having a magnetic moment μ, inside a box of volume V, with a magnetic field B along the z-axis. The energy of the atoms is given by

$$E = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} - \sum_{i=1}^{N} \mu B \cos\theta_i,$$

   where θ_i is the angle that the magnetic moment of the i'th atoms makes with the magnetic field. Evaluate the average magnetization of the gas at a temperature T.